



# *Everyday Mathematics*

## Partial-Quotients Algorithm (Focus Algorithm)



# Partial-Quotients Algorithm

We use **partial-quotients division** to solve problems such as:

$$42 \overline{)815}$$

The problem is read, “815 divided by 42.”

815 is the **dividend**, the number we are dividing.

42 is the **divisor**, the number we are dividing by.

# Partial-Quotients Algorithm

Partial-quotients division involves:

- Breaking the **dividend** into parts,
- Finding **multiples** of the **divisor**,
- Finding partial **quotients**, and
- Finding the **sum** of the partial quotients.

$$\begin{array}{r} \text{divisor} \\ \downarrow \\ 42 \overline{)815} \\ \uparrow \\ \text{dividend} \end{array}$$

## Definitions:

A **multiple** of a number is the product of that number and some counting number (1, 2, 3, 4, 5, etc.).

The **product** is the answer to a multiplication problem.

The **quotient** is the answer to a division problem.

The **sum** is the answer to an addition problem.

# Partial-Quotients Algorithm

We will solve:  $42 \overline{)815}$

We can solve this problem by thinking about:

How many [42s] are in 815?

*Or*

What times 42 will equal 815?

# Partial-Quotients Algorithm

To find  $815 \div 42$ , begin by thinking about easy *multiples* of 42:

$$1 \times 42 = 42$$

$$10 \times 42 = 420$$

Let's use these two facts to generate some others:

$$2 \times 42 = 84 \quad [\text{double } 1 \times 42]$$

$$4 \times 42 = 168 \quad [\text{double } 2 \times 42]$$

$$5 \times 42 = 210 \quad [\text{take } \frac{1}{2} \text{ of } 10 \times 42]$$

$$15 \times 42 = 630 \quad [\text{solve } 3 \times (5 \times 42)]$$

$$20 \times 42 = 840 \quad [\text{double } 10 \times 42 \text{ or solve } 10 \times (42 \times 2)]$$

Our dividend of 815 is between 630 and 840,  
so we can stop here.

# Partial-Quotients Algorithm

Set up the notation  
for using the partial-  
quotients algorithm.

$$42 \overline{)815}$$

Draw a line to the right  
of the problem →

Write partial quotients here:



# Partial-Quotients Algorithm

How many [42s] are in 815?

Recall the multiples we came up with:

$$15 \times 42 = 630$$

$$20 \times 42 = 840$$

$$42 \overline{)815}$$

Partial quotients



840 is greater than 815, so there are fewer than 20 [42s] in 815. But 630 is less than 815, which means that there are at least 15 [42s] in 815. So let's use 15 as our first partial quotient.

# Partial-Quotients Algorithm

The first partial quotient is 15.

Record 15 to the right of the problem.

$$15 \times 42 = 630$$

Record 630 below the dividend.

$$\begin{array}{r} 42 \overline{) 815} \\ 630 \end{array}$$

Partial quotients

↓  
15



# Partial-Quotients Algorithm

Now subtract to  
find the difference.

185 is the  
remainder.

$$\begin{array}{r} 42 \overline{) 815} \\ - 630 \\ \hline 185 \end{array}$$

Partial quotients

↓  
15

# Partial-Quotients Algorithm

How many [42s] are  
in 185?

Recall the multiples we  
came up with:

$$4 \times 42 = 168$$

$$5 \times 42 = 210$$

$$\begin{array}{r} 42 \overline{) 815} \\ \underline{630} \\ 185 \end{array}$$

Partial quotients

↓  
15

210 is greater than 185, so there are fewer than 5 [42s] in 185. But 168 is less than 185, which means that there are at least 4 [42s] in 185. So let's use 4 as our second partial quotient.

# Partial-Quotients Algorithm

The second partial quotient is 4.

Record 4 to the right of the problem.

$$4 \times 42 = 168$$

Record 168 below the 185.

$$\begin{array}{r} 42 \overline{) 815} \\ - \underline{630} \\ 185 \\ 168 \end{array}$$

Partial quotients

↓  
15

4

# Partial-Quotients Algorithm

Now subtract to  
find the difference.

17 is the new  
remainder. Since  
17 is less than 42,  
we are done  
subtracting  
multiples of the  
divisor.

$$\begin{array}{r} 42 \overline{) 815} \\ \underline{630} \\ 185 \\ - 168 \\ \hline 17 \end{array}$$

Partial quotients

↓  
15

4

# Partial-Quotients Algorithm

Now add  
the partial  
quotients.

The quotient  
is 19 R17.

	Partial quotients
$\begin{array}{r} 42 \overline{) 815} \\ \underline{630} \\ 185 \\ \underline{168} \\ 17 \end{array}$	$\begin{array}{r} \downarrow \\ 15 \\ + 4 \\ \hline 19 \end{array}$

# Partial-Quotients Algorithm

$$815 \div 42 \rightarrow 19 \text{ R}17$$

Note that when children use **partial-quotients division** to solve a division problem, they have an opportunity to practice a variety of skills related to developing number sense and algebraic reasoning.

*These skills include:*

- *Using equivalent names for numbers when breaking down the divisor,*
- *Using multiples to solve the problem,*
- *Practicing doubling and halving (if using a fact strategy such as the one in this presentation),*
- *Using all four operations — addition, subtraction, multiplication, and division.*